Coverage Control for Multi-Robot Teams with Heterogeneous Sensing Capabilities Using Limited Communications*

María Santos and Magnus Egerstedt[†]

Abstract— **This paper presents a coverage algorithm for multi-robot systems where the robots are equipped with qualitatively different sensing modalities. Unlike previous approaches to the problem of coverage for teams with heterogeneous sensing capabilities, in this paper the robots have access to information about their neighbors' specific sensor modalities. This knowledge affords the ability of ensuring that no robot is tasked with covering features in a region without the required sensing modalities. With this information, a robot can determine which of its neighbors it should coordinate with to cover the environmental features in a region while ignoring robots which are not equipped with that particular sensory capability. We derive a distributed control algorithm that allows the robots to move in a direction of descent relative to a novel locational cost, in order to minimize it. The performance of the algorithm is evaluated on a real robotic platform.**

I. INTRODUCTION

The problem of coverage for multi-robot teams deals with the positioning of a team of robots in a domain of interest such that the environmental features in the domain are monitored by at least one of the robots in the team [1], [2], [3], [4]. Many different aspects of the coverage problem have been explored in the literature, including limitations on the robots' motion performance [5], [6], variations on the environmental features present in the domain [7], or geometric characteristics of the sensor footprint [8], [9], among others. However, the robots are often interchangeable in terms of the kind of features they monitor, i.e., all the robots in the team are equipped with the same sensing modalities, thus being able to detect the same events in the domain, even when differences arise between the robots in terms of performance [4], [10].

In complex environments, a multi-robot team may need to simultaneously monitor multiple types of features (e.g. radiation, humidity, temperature [11]), which require a mixture of sensing capabilities too extensive to be designed into a single robot [12]. As an alternative, each robot may be equipped with a subset of those sensors as long as, collectively, the team has all the sensor modalities needed to monitor the collection of features in the domain. However, in that case, the formulation of the coverage control algorithm needs to account for the sensing capabilities of each of the robots in the team. Past work on heterogeneous coverage problems with qualitatively different sensors [13] has focused on how those robots should coordinate without knowledge of the specific sensor modalities of the neighboring robots. In this paper, we investigate how the performance of the coverage algorithm can be improved through the addition of knowledge of the sensing modalities.

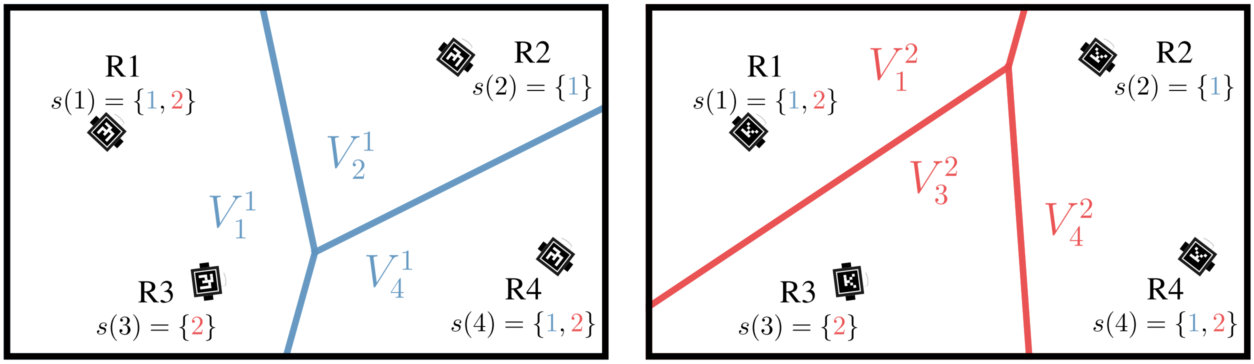
In particular, we consider a team of N planar robots equipped with a set of sensor modalities, $\mathcal{S}$, each capable of monitoring a particular environmental feature present in the domain $\mathcal{D} \subset \mathbb{R}^2$. Analogously to [13], given that the importance of a region may vary depending on the feature being tracked, we consider a family of density functions, $\phi_j : \mathcal{D} \to [0, \infty)$, with $\phi_j(q)$ representing the importance of a point $q \in \mathcal{D}$ according to sensor $j \in \mathcal{S}$. By allowing Robot i to know which of its neighbors share some of its sensing modalities, it can determine which robots it should share responsibility with when covering each of the associated density functions, $\phi_j, j \in s(i)$. We demonstrate that making the information of its neighbors' sensors available to each robot allows the multi-robot team to ensure that no robot is tasked with covering a region without the required sensing modalities.

Providing each robot with information about its neighbors' sensing capabilities presents a major advantage of this approach when compared to [13], where the domain is partitioned into regions of dominance according to a Voronoi tessellation [14]. Given that a Voronoi partition generated by all the robots in the team does not take into account their individual sensory capabilities, it is possible for a robot to be left in charge of covering an area without the sensors to monitor the features in it. In order to avoid leaving any region unsurveilled, an additional term that evaluates the overall domain performance of each robot with respect to its sensors was added in [13]. In this paper, the information about the sensing modalities of each robot is taken into account when determining the regions of dominance, which are calculated separately for each sensor type. Thus, it is ensured that the responsibility of covering an area is only given to a robot if it has the appropriate sensors and the evaluation of an overall domain performance term as in [13] becomes redundant.

This paper presents a distributed control law for a multi-robot team with heterogeneous sensing capabilities where each robot has knowledge about the sensor capabilities of its neighbors. The outline of the paper is as follows: In Section II we present a new locational cost that evaluates the performance of each robot with respect to its sensors' density functions over generalized regions of dominance. The derivation of the gradient to the locational cost is shown in Section III, along with the distributed control law that minimizes the cost. The experiments conducted on real

*This work was supported in part by the Army Research Lab under Grant #W911NF-17-0181 and in part by "la Caixa" Banking Foundation.

[†]The authors are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, Georgia, USA {maria.santos,magnus}@gatech.edu



(a) Regions of dominance with respect to Sensor 1.

(b) Regions of dominance with respect to Sensor 2.

Fig. 1. Regions of dominance for four neighboring robots with respect to sensor 1, (a), and sensor 2, (b). In this example, the sensing performance in a point $q \in \mathcal{D}$ degrades with the square of the distance to the robot: $f_j(d_j(p_i, q)) = \|p_i - q\|^2$, $i \in \{1, \ldots, 4\}$, $j \in \{1, 2\}$. For each sensor j, the resulting regions of dominance, V_i^j, are Voronoi cells generated by those robots equipped with the sensor.

robotic platforms are included in Section IV. Section V concludes the paper.

II. HETEROGENEOUS LOCATIONAL COST

Given a multi-robot system in charge of covering an area, a fundamental question one can ask is what constitutes a good spatial allocation of the robots in the team. Analogously to other multi-robot coverage problems [1], [3], [4], [7], we are on the quest of defining a locational cost, $\mathcal{H}(p)$, that quantifies the quality of the coverage as a function of the positions of the robots in the team.

A common strategy to evaluate the system performance when covering a single density function, $\phi : \mathcal{D} \to [0, \infty)$, is to divide the domain $\mathcal{D}$ into regions of dominance, $\{V_1, \ldots, V_N\}$, such that Robot i is responsible for the coverage of region V_i. Then, the quality of coverage obtained in region V_i can be encoded through the cost

$$h_i(p_i, V_i) = \int_{V_i} f(d(p_i, q))\phi(q)\,\mathrm{d}q, \tag{1}$$

where $d : \mathcal{D} \times \mathcal{D} \to \mathbb{R}$ measures the distance between the robots and the points in the domain and $f : \mathbb{R} \to \mathbb{R}$ a smooth strictly increasing function over the range of d that measures the degradation of sensing performance with distance [4], [14].

Analogously, in situations where robots have qualitatively different sensing modalities, we can evaluate the quality of coverage performed by Robot i with respect to sensor j as the cost

$$h_i^j(p_i, V_i^j) = \int_{V_i^j} f_j(d_j(p_i, q))\phi_j(q)\,\mathrm{d}q, \tag{2}$$

where V_i^j is the region of dominance of Robot i with respect to sensor j, and the distance and degradation functions, d_j and f_j, may be dependent on the type of sensor. Note that, for a system where the robots are equipped with different sensors, the region that Robot i is responsible for with respect to sensor j, V_i^j, can differ from the region to be monitored with respect to sensor k, V_i^k, $j, k \in s(i)$, depending on the sensor equipments of Robot i's neighbors. An example of this for a four-robot, two-sensor scenario is shown in Fig. 1, where we can observe that the regions of dominance for Sensor 1, V_i^1, differ from those of Sensor 2, V_i^2, given that some robots are not equipped with those sensors and, thus, ignored when computing the corresponding regions of dominance.

In order to calculate the cost in (2), we need to determine what are the regions of dominance, V_i^j. For sensor j and its corresponding density, ϕ_j, a natural choice is to define the boundaries of V_i^j based on the position of those robots in the team equipped with sensor j that are closest to Robot i (in terms of the distance function d_j). If we denote as $\mathcal{N}^j$ the set of robots equipped with sensor j,

$$\mathcal{N}^j = \{i \in \mathcal{N} \mid j \in s(i)\}, \quad \mathcal{N} = \{1, \ldots, N\},$$

then the region of dominance of Robot i with respect to Sensor $j \in s(i)$ can be defined as a function of the positions of the robots according to the so-called *nearest-neighbor rule* [15],

$$V_i^j(p) = \{q \in \mathcal{D} \mid d_j(p_i, q) \le d_j(p_k, q), \forall k \in \mathcal{N}^j\}. \tag{3}$$

The regions of dominance for Sensor j therefore correspond to the Voronoi partition generated by the robots in $\mathcal{N}^j$. Note that, if i is the only robot equipped with Sensor j, then the robot is in charge of covering the whole domain: $V_i^j = \mathcal{D}$.

With the regions of dominance defined, we can calculate the cost given by (2) for all the robots and all their sensors. With this information, the performance of the multi-robot team can be encoded through the *heterogeneous locational cost*,

$$\mathcal{H}(p) = \sum_{j \in \mathcal{S}} \sum_{i \in \mathcal{N}^j} \int_{V_i^j(p)} f_j(d_j(p_i, q))\phi_j(q)\,\mathrm{d}q, \tag{4}$$

with a lower value of the cost corresponding to a better coverage of the domain.

The proposed heterogeneous locational cost has two significant advantages when compared to the cost in [13]: On the one hand, the boundaries between the regions of dominance, $\partial V_i^j, i \in \mathcal{N}$, are defined with respect to the same density function, $\phi_j, j \in \mathcal{S}$, so that the cancellations that occur when applying Leibniz rule in the homogeneous case [1], will also take place when calculating the gradient of (4). On the other hand, when covering the density ϕ_j, a robot equipped with sensor j will only relinquish the responsibility over an area in the domain to a neighboring robot if the latter is also equipped with sensor j and can perform a better coverage of such area (according to the functions d_j and f_j). This strategy thus ensures that no robot is ever in charge of covering a density over an area without the corresponding sensors and, as a result, there is no need to evaluate the performance of a robot over the whole domain, as was done in [13].

III. GRADIENT DESCENT

Having defined a locational cost that evaluates the quality of the coverage performed by the multi-robot team, we need to establish how the robots should move in the domain in order to minimize it. A standard approach to minimize the cost is to let each robot move in a direction of descent of the gradient, that is,

$$\dot{p}_i = -\gamma_i(p)\frac{\partial \mathcal{H}(p)^T}{\partial p_i}, \quad i \in \mathcal{N},$$

with $\gamma_i(p) > 0$ a gain for Robot i, which can depend on the position of the robots.

In order to calculate the derivative of the cost with respect to the position of Robot i, we need only consider the terms associated with the sensor equipment of Robot i, $s(i)$,

$$\frac{\partial \mathcal{H}}{\partial p_i} = \frac{\partial}{\partial p_i}\left(\sum_{j\in s(i)}\sum_{k\in\mathcal{N}^j}\int_{V_k^j(p)} f_j(d_j(p_k,q))\phi_j(q)\,\mathrm{d}q\right), \tag{5}$$

given that the remainder terms in the first summation in (4) do not depend on p_i. Subsequently, for each sensor $j \in s(i)$, we can break down the expression in (5) in terms of the contribution of Robot i, its Delaunay neighbors with respect to such sensor, $\mathcal{N}_i^j$, and the rest of the robots in $\mathcal{N}^j$,

$$\frac{\partial \mathcal{H}}{\partial p_i} = \frac{\partial}{\partial p_i}\left(\sum_{j\in s(i)}\int_{V_i^j(p)} f_j(d_j(p_i,q))\phi_j(q)\,\mathrm{d}q\right) \tag{6}$$

$$+\frac{\partial}{\partial p_i}\left(\sum_{j\in s(i)}\sum_{k\in\mathcal{N}_i^j}\int_{V_k^j(p)} f_j(d_j(p_k,q))\phi_j(q)\,\mathrm{d}q\right) \tag{7}$$

$$+\frac{\partial}{\partial p_i}\left(\sum_{j\in s(i)}\sum_{\substack{k\in\mathcal{N}^j\\ k\notin\mathcal{N}_i^j\cup\{i\}}}\int_{V_k^j(p)} f_j(d_j(p_k,q))\phi_j(q)\,\mathrm{d}q\right), \tag{8}$$

where the term (8) is zero since it does not depend on p_i. Analogously to what was done in [13], we need to apply Leibniz integral rule [16] to calculate the derivative of the first two terms.

Lemma 1 (Leibniz Integral Rule [16])**.** *Let $\Omega(p)$ be a region that depends smoothly on p such that the unit outward normal vector $n(p)$ is uniquely defined almost everywhere on the boundary $\partial\Omega(p)$. Let*

$$F = \int_{\Omega(p)} f(q)\,\mathrm{d}q.$$

Then,

$$\frac{\partial F}{\partial p} = \int_{\partial\Omega(p)} f(q)n(q)^T\frac{\partial q}{\partial p}\,\mathrm{d}q,$$

where $\int_{\partial\Omega(p)}$ denotes the line integral over the boundary of $\Omega(p)$.

The boundary of the region of dominance of Robot i with respect to sensor j, $\partial V_i^j(p)$, depends on p_i when a neighboring Robot k is also equipped with sensor j, $k \in \mathcal{N}_i^j$. In that case we can denote as $\partial V_{ik}^j(p)$ the boundary between Robots i and k, with the points on that boundary satisfying the equality condition in the nearest-neighbor rule,

$$\partial V_{ik}^j(p) = \{q \in \mathcal{D} \mid d_j(p_i,q) = d_j(p_k,q), k \in \mathcal{N}_i^j\}. \tag{9}$$

Using this notation, we can write the derivative of (6) as

$$\sum_{j\in s(i)}\int_{V_i^j(p)}\frac{\partial f_j(d_j(p_i,q))}{\partial p_i}\phi_j(q)\,\mathrm{d}q$$
$$+\sum_{j\in s(i)}\sum_{k\in\mathcal{N}_i^j}\int_{\partial V_{ik}^j(p)} f_j(d_j(p_i,q))\phi_j(q)n_{ik}^j(q)^T\frac{\partial q}{\partial p_i}\,\mathrm{d}q, \tag{10}$$

where $n_{ik}(q)$, $q \in \partial V_{ik}^j(p)$, denotes the unit outward normal vector on the boundary between Robots i and k. Similarly, the derivative in (7) becomes,

$$-\sum_{j\in s(i)}\sum_{k\in\mathcal{N}_i^j}\int_{\partial V_{ik}^j(p)} f_j(d_j(p_k,q))\phi_j(q)n_{ik}^j(q)^T\frac{\partial q}{\partial p_i}\,\mathrm{d}q, \tag{11}$$

since the integrand of (7) does not depend on p_i and $n_{ik}(q) = -n_{ki}(q), \forall q \in \partial V_{ik}^j(p)$.

The integral terms over the boundaries in (10) and (11) cancel given that the points on the boundary, $q \in \partial V_{ik}^j(p)$, satisfy the condition in (9) and therefore $f_j(d_j(p_i,q)) = f_j(d_j(p_k,q))$. Thus, the gradient of $\mathcal{H}(p)$ becomes,

$$\frac{\partial \mathcal{H}}{\partial p_i} = \sum_{j\in s(i)}\int_{V_i^j(p)}\frac{\partial f_j(d_j(p_i,q))}{\partial p_i}\phi_j(q)\,\mathrm{d}q. \tag{12}$$

Letting Robot i follow a negative gradient descent establishes the following control law.

Theorem 1 (Heterogeneous Gradient Descent)**.** *Let Robot i, with planar position p_i, evolve according to the control law $\dot{p}_i = u_i$, where*

$$u_i = -\kappa\sum_{j\in s(i)}\int_{V_i^j(p)}\frac{\partial f_j(d_j(p_i,q))}{\partial p_i}\phi_j(q)\,\mathrm{d}q \tag{13}$$

Then, as $t \to \infty$, the robots will converge to a critical point of the heterogeneous location cost in (4) *under a positive gain $\kappa > 0$.*

TABLE I

SENSOR CONFIGURATION AND DENSITY PARAMETERS FOR THE EXPERIMENTS

Experiment	Team Sensors	Robot Sensors	Density Functions (in cm)
1	$\mathcal{S} = \{1\}$	$s(i) = \{1\}, i \in \mathcal{N}$	$\phi_1(q) = \mathcal{G}(q, [0,0]^T)$
2	$\mathcal{S} = \{1, 2\}$	$s(i) = \{1\}, i \in \{1, \dots, 5\}$ $s(i) = \{2\}, i \in \{6, \dots, 10\}$	$\phi_1(q) = \mathcal{G}(q, [-50,0]^T)$ $\phi_2(q) = \mathcal{G}(q, [50,0]^T)$
3	$\mathcal{S} = \{1, \dots, 10\}$	$s(i) = i, i \in \mathcal{N}$	$\phi_1(q) = \mathcal{G}([-55,0]^T)$ $\quad \phi_2(q) = \mathcal{G}(q, [55,0]^T)$ $\phi_3(q) = \mathcal{G}(q, [-38,38]^T)$ $\quad \phi_4(q) = \mathcal{G}(q, [38,-38]^T)$ $\phi_5(q) = \mathcal{G}(q, [-38,-38]^T)$ $\quad \phi_6(q) = \mathcal{G}(q, [38,38]^T)$ $\phi_7(q) = \mathcal{G}(q, [-20,0]^T)$ $\quad \phi_8(q) = \mathcal{G}(q, [20,0]^T)$ $\phi_9(q) = \mathcal{G}(q, [0,40]^T)$ $\quad \phi_{10}(q) = \mathcal{G}(q, [0,-40]^T)$
4	$\mathcal{S} = \{1, \dots, 5\}$	$s(1) = \{1\}$ $\quad s(2) = \{2\}$ $s(3) = \{3\}$ $\quad s(4) = \{4\}$ $s(5) = \{5\}$ $\quad s(6) = \{1\}$ $s(7) = \{2\}$ $\quad s(8) = \{3\}$ $s(9) = \{4\}$ $\quad s(10) = \{5\}$	$\phi_1(q) = G(q, [-40,-40]^T)$ $\phi_2(q) = G(q, [-40,40]^T)$ $\phi_3(q) = \mathcal{G}(q, [0,0]^T)$ $\phi_4(q) = \mathcal{G}(q, [40,40]^T)$ $\phi_5(q) = \mathcal{G}(q, [40,-40]^T)$
5	$\mathcal{S} = \{1, \dots, 4\}$	$s(1) = \{1, \dots, 4\}$ $\quad s(2) = \{1\}$ $s(3) = \{1, 2\}$ $\quad s(4) = \{1, 3\}$ $s(5) = \{1, 2, 4\}$ $\quad s(6) = \{1\}$ $s(7) = \{1, 2, 3\}$ $\quad s(8) = \{1\}$ $s(9) = \{1, 2, 4\}$ $\quad s(10) = \{1, 3\}$	$\phi_1(q) = G(q, [0,0]^T)$ $\phi_2(q) = \mathcal{U}(\mathcal{D})$ $\phi_3(q) = \mathcal{G}(q, [-60,0]^T)$ $\phi_4(q) = \mathcal{G}(q, [-0,40]^T)$ $+\mathcal{G}(q, [0,-40]^T)$
6	$\mathcal{S} = \{1, \dots, 3\}$	$s(1) = \{1, 2, 3\}$ $\quad s(2) = \{1, 2\}$ $s(3) = \{2, 3\}$ $\quad s(4) = \{1, 3\}$ $s(5) = \{1\}$ $\quad s(6) = \{1\}$ $s(7) = \{2\}$ $\quad s(8) = \{2\}$ $s(9) = \{3\}$ $\quad s(10) = \{3\}$	$\phi_1(q) = \mathcal{G}(q, [0,40])$ $\phi_2(q) = \mathcal{G}(q, [-34,-20]^T)$ $\phi_3(q) = \mathcal{G}(q, [34,20]^T)$

Proof. The form of the gradient is given in (12). Consider the locational cost $\mathcal{H}(p) > 0$ as a candidate function to prove convergence to a critical point. The total derivative of the locational cost,

$$\frac{d\mathcal{H}(p)}{dt} = \sum_{i \in \mathcal{N}} \frac{\partial \mathcal{H}(p)}{\partial p_i} \dot{p}_i = -\kappa \left\| \frac{\partial \mathcal{H}(p)}{\partial p}^T \right\|^2 \leq 0. \tag{14}$$

The total derivative in (14) is zero if $\partial \mathcal{H}(p)/\partial p = 0$, in which case the control law becomes $\dot{p}_i = 0$. By LaSalle's invariance principle, the positions of the multi-robot system, p will converge to the largest invariant set contained in the set of all points such that $d\mathcal{H}/dt = 0$, that is, the critical points to the heterogeneous locational cost in (4). $\square$

Note that, when the function d_j is defined as the Euclidean distance and the degradation function, f_j, takes the square of that distance as in [1], [13], that is,

$$f_j(d_j(p_i, q)) = \|p_i - q\|^2, \ \forall i \in \mathcal{N}, j \in \mathcal{S},$$

then the control law in Theorem 1 becomes,

$$\dot{p}_i = 2 \sum_{j \in s(i)} m_i^j (c_i^j - p_i), \tag{15}$$

where m_i^j and c_i^j are defined as the *heterogeneous mass* and *center of mass* of Robot i with respect to sensor j,

$$m_i^j = \int_{V_i^j(p)} \phi_j(q) \, \mathrm{d}q, \quad c_i^j = \frac{\int_{V_i^j(p)} q \phi_j(q) \, \mathrm{d}q}{m_i^j}. \tag{16}$$

Therefore, using the square of the Euclidean distance as the performance measure results in a controller that makes each robot move according to a weighted sum where each summation term corresponds to performing a continuous-time Lloyd descent as in [1] over the region of dominance corresponding to each of the sensors of the robot.

IV. EXPERIMENTAL RESULTS

The proposed controller is implemented on a team of ten GRITSBots [17], which are differential-drive robots available at the Robotarium [18], a remotely accessible multi-robot testbed where the code is uploaded via web interface and the experimental data can be retrieved after the experiment is finalized. On each iteration, the Robotarium interface provides information about the position and orientation of the robots in the team and allows to specify the linear and angular velocities to be executed by the robots.

An implicit assumption behind the controller in Theorem 1 is that each robot can move according to single integrator dynamics, $\dot{p}_i = u_i, \forall i \in \mathcal{N}$. However, given that GRITS-Bots cannot directly execute single integrator dynamics, the control commands produced by (13) must be converted into inputs executable by these differential drive robots, i.e., linear and angular velocity commands. Let $p_i = (x_i, y_i)^T$ be the planar position of Robot i and θ_i, its orientation. The movement of a differential-drive robot can be modeled according to the unicycle dynamics,

$$\dot{x}_i = v_i \cos\theta_i, \quad \dot{y}_i = v_i \sin\theta_i, \quad \dot{\theta}_i = \omega_i,$$

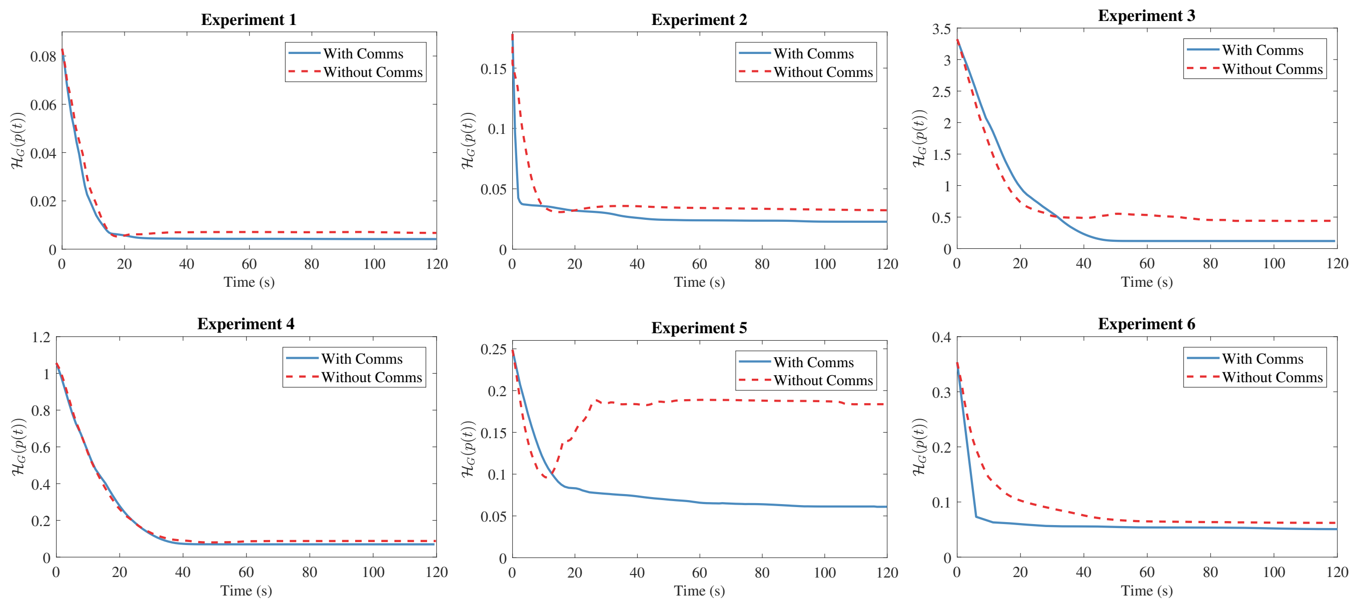


Fig. 2. Evolution of the global performance cost, $\mathcal{H}_G$, for the proposed gradient descent algorithm in Theorem 1 and the controller without communications in [13]. We can see how, throughout the six experiments, the cost attained by the proposed controller is consistently equal or smaller than the one attained by the other controller. The performance difference is particularly acute in the case of Experiment 5, where the controller that does not consider communications remains in a critical point of the cost that does not correspond to a good overall coverage of the domain.

where v_i and ω_i are the linear and angular velocities executed by the robots. The single integrator dynamics in (13) can be approximated using a model similar to the one in [1],

$$v_i = k_v[\cos\theta_i \quad \sin\theta_i]\dot{p}_i,$$
$$\omega_i = k_\omega \arctan\left(\frac{[-\sin(\theta_i) \quad \cos(\theta_i)]\dot{p}_i}{[\cos(\theta_i) \quad \sin(\theta_i)]\dot{p}_i}\right),$$

with k_v and k_ω positive gains.

In order to evaluate the performance of the proposed algorithm, we compare its performance to the descent flow algorithm presented in [13], which does not consider communications about sensor modalities. To this end, we define a baseline cost function which captures the team's performance when global information is available, that is, the position and sensor equipment of all the robots. According to the nearest-neighbor rule [15], a point in the domain is best covered with respect to ϕ_j when its coverage is assigned to the closest robot equipped with sensor j, in which case the optimal coverage of the density ϕ_j is be given by,

$$\int_{\mathcal{D}} \min_{i\in\mathcal{S}^j} f_j(d_j(p_i,q))\phi_j(q)\,\mathrm{d}q,$$

with $\mathcal{S}^j$ the set of all robots with sensor modality j,

$$\mathcal{S}^j = \{i \in \mathcal{N} \mid j \in s(i)\},\ j \in \mathcal{S}.$$

The performance of the team with respect to all the density functions, $\phi_j, j \in \mathcal{S}$, can be therefore obtained by adding the cost pertaining to each of the sensors, i.e.,

$$\mathcal{H}_G = \sum_{j\in\mathcal{S}} \int_{\mathcal{D}} \min_{i\in\mathcal{S}^j} f_j(d_j(p_i,q))\phi_j(q)\,\mathrm{d}q, \tag{17}$$

where G denotes the *global performance* of the team.

We use the cost in (17) as a baseline to compare the control law proposed in Theorem 1 to the controller in [13]. In order to provide a fair assessment of their performance, the distance and degradation functions assigned to all the sensors in this section are defined as,

$$f_j(d_j(p_i,q)) = \|p_i - q\|^2,\ \forall j \in \mathcal{S},\ i \in \mathcal{N},$$

given that the square of the Euclidean distance is the only sensing performance measured considered in [13].

The experiment consists on six different configurations of team sensors, $\mathcal{S}$; robot sensors, $s(i), i \in \mathcal{N}$; and of the corresponding densities, $\phi_j, j \in \mathcal{S}$. The sensing capabilities of the robots are simulated based on the pose information provided by the tracking system, from which the corresponding sensing information is provided to each robot. The sensor and density configurations used for each experiment are included in Table I. Except for a uniform density, $\mathcal{U}(\mathcal{D})$, used in Exp. 5, all the density functions are bivariate normal distributions as used in [1], [3], where the covariance matrix, Σ, is kept constant for all the experiments. The notation used in Table I indicates the location of the mean,

$$\mathcal{G}(q,\mu) = \frac{1}{2\pi\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2}(q-\mu)^T\Sigma^{-1}(q-\mu)\right).$$

The evolution of the global performance cost for the six experiments is shown in Fig. 2, where, for all the cases, the critical point attained by the proposed controller corresponds to a lower value of the performance measure in (17) with respect to the controller without communications.

The final spatial configurations attained by both algorithms correspond to critical points of their corresponding locational costs. Final allocations of the team for Experiments 2, 4 and 6, run under identical initial conditions, are compared in Fig. 3. In Experiment 2, the team is in charge of covering two density functions, with half of the robots assigned to each of them and no robot being in charge of both. Without communications (Fig. 3a), most of the robots are located close to their area of higher density, with the exception of a pair that establish a Voronoi boundary regardless of them not sharing any sensing capabilities. This situation is not observed running proposed algorithm, Fig. 3b, given that boundaries are only established among robots equipped with the same sensors.

In Experiment 4, the team is divided in pairs to cover five different densities. As shown in Fig. 3c, having no communications about their sensing capabilities again results in two robots establishing a boundary with a neighbor without common sensors. Given that the gradient points each of these robots perpendicularly to their boundary, the team settles in this critical point. In contrast, the proposed algorithm (Fig. 3d), achieves a satisfactory spatial configuration where the robots are located in areas of high interest according to their sensors. In fact, the boundary of the regions of dominance for each pair crosses the corresponding density area in the middle, dividing the domain in two areas containing the same mass. As for Experiment 6, several robots are equipped with multiple sensing capabilities. In this case, both algorithms successfully place the robots with shared sensors in between the regions of high density, while the robots equipped with only one sensor occupy more dedicated positions with respect to their densities.

In general, defining the regions of dominance considering only those neighbors equipped with common sensors constitutes a major advantage of the proposed algorithm, since the robots can overcome locations populated by robots with which they not share coverage responsibilities in order to reach areas of higher density with respect to their sensors, where they coordinate with robots that also share the same responsibilities.

V. CONCLUSIONS

In this paper, we introduced a coverage control algorithm for multi-robot teams with heterogeneous sensing capabilities where the robots have information about the sensor modalities of their neighbors. With this information flow, the regions of dominance of a robot were defined independently for each of its sensors as a function of its neighbors sensor equipments. We presented a new locational cost that evaluates the performance of the team when covering the different density functions over such regions of dominance and derived a distributed gradient descent algorithm that allows the multi-robot system to achieve a critical point of the locational cost. The proposed gradient descent algorithm was evaluated through a series of experiments conducted on a team of differential-drive robots and performance compared to another algorithm for teams with heterogeneous sensing capabilities that does not consider communications among the robots. The experiments suggest that incorporating communications among the team members indeed improves the quality of coverage on the heterogeneous sensing capabilities scenario, as the proposed algorithm achieved better values of the performance metric for a number of different density and sensor configurations.

REFERENCES

[1] J. Cortes, S. Martinez, T. Karatas, and F. Bullo, "Coverage control for mobile sensing networks," *IEEE Transactions on Robotics and Automation*, vol. 20, no. 2, pp. 243–255, Apr. 2004.

[2] F. Bullo, J. Cortes, and S. Martinez, *Distributed Control of Robotic Networks: A Mathematical Approach to Motion Coordination Algorithms*, ser. Applied Mathematics Series. Princeton, NJ, USA: Princeton University Press, 2009.

[3] M. Schwager, J. Mclurkin, and D. Rus, "Distributed coverage control with sensory feedback for networked robots," in *in Proceedings of Robotics: Science and Systems*, 2006.

[4] L. C. A. Pimenta, V. Kumar, R. C. Mesquita, and G. A. S. Pereira, "Sensing and coverage for a network of heterogeneous robots," in *IEEE Conference on Decision and Control*, Cancún, México, Dec. 2008, pp. 3947–3952.

[5] A. Pierson, L. C. Figueiredo, L. C. A. Pimenta, and M. Schwager, "Adapting to performance variations in multi-robot coverage," in *2015 IEEE International Conference on Robotics and Automation (ICRA)*, Seattle, WA, May 2015, pp. 415–420.

[6] A. Kwok and S. Martinez, "Deployment algorithms for a power-constrained mobile sensor network," in *2008 IEEE International Conference on Robotics and Automation*, May 2008, pp. 140–145.

[7] S. G. Lee, Y. Diaz-Mercado, and M. Egerstedt, "Multi-robot control using time-varying density functions," *IEEE Transactions on Robotics*, vol. 31, no. 2, pp. 489–493, Apr. 2015.

[8] A. Gusrialdi, T. Hatanaka, and M. Fujita, "Coverage control for mobile networks with limited-range anisotropic sensors," in *2008 47th IEEE Conference on Decision and Control*, Dec 2008, pp. 4263–4268.

[9] B. Hexsel, N. Chakraborty, and K. Sycara, "Coverage control for mobile anisotropic sensor networks," in *2011 IEEE International Conference on Robotics and Automation*, May 2011, pp. 2878–2885.

[10] O. Arslan and D. E. Koditschek, "Voronoi-based coverage control of heterogeneous disk-shaped robots," in *2016 IEEE International Conference on Robotics and Automation (ICRA)*, Stockholm, Sweden, May 2016, pp. 4259–4266.

[11] J. P. Grotzinger, J. Crisp, A. R. Vasavada, R. C. Anderson, C. J. Baker, R. Barry, D. F. Blake, P. Conrad, K. S. Edgett, B. Ferdowski, R. Gellert, J. B. Gilbert, M. Golombek, J. Gómez-Elvira, D. M. Hassler, L. Jandura, M. Litvak, P. Mahaffy, J. Maki, M. Meyer, M. C. Malin, I. Mitrofanov, J. J. Simmonds, D. Vaniman, R. V. Welch, and R. C. Wiens, "Mars science laboratory mission and science investigation," *Space Science Reviews*, vol. 170, no. 1, pp. 5–56, Sep 2012.

[12] L. E. Parker, "Heterogeneous multi-robot cooperation," Ph.D. dissertation, Cambridge, MA, USA, 1994.

[13] M. Santos, Y. Diaz-Mercado, and M. Egerstedt, "Coverage control for multirobot teams with heterogeneous sensing capabilities," *IEEE Robotics and Automation Letters*, vol. 3, no. 2, pp. 919–925, April 2018.

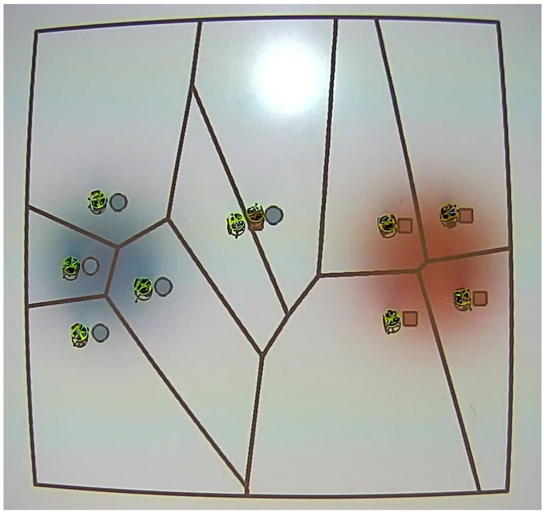
[14] A. Okabe, B. Boots, K. Sugihara, and S. N. Chiu, *Spatial Tessellations: Concepts and Applications of Voronoi Diagrams*. New York, NY, USA: John Wiley & Sons, Inc., 2000.

[15] F. Aurenhammer, "Voronoi diagrams – a survey of a fundamental geometric data structure," *ACM Comput. Surv.*, vol. 23, no. 3, pp. 345–405, Sept. 1991.

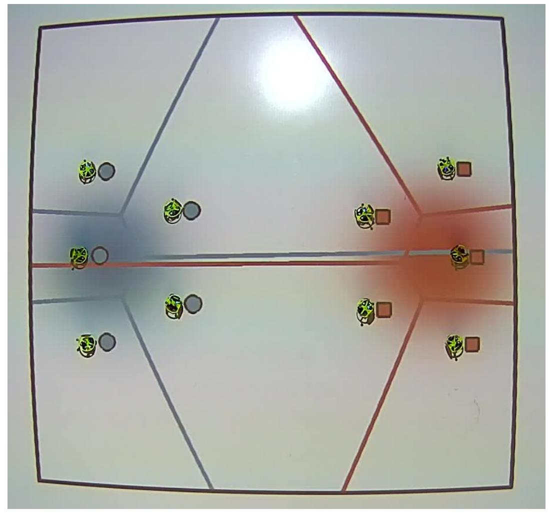
[16] Q. Du, V. Faber, and M. Gunzburger, "Centroidal voronoi tessellations: Applications and algorithms," *SIAM Review*, vol. 41, no. 4, pp. 637–676, Dec. 1999.

[17] D. Pickem, M. Lee, and M. Egerstedt, "The GRITSBot in its natural habitat – a multi-robot testbed," in *2015 IEEE International Conference on Robotics and Automation (ICRA)*, Seattle, WA, May 2015, pp. 4062–4067.

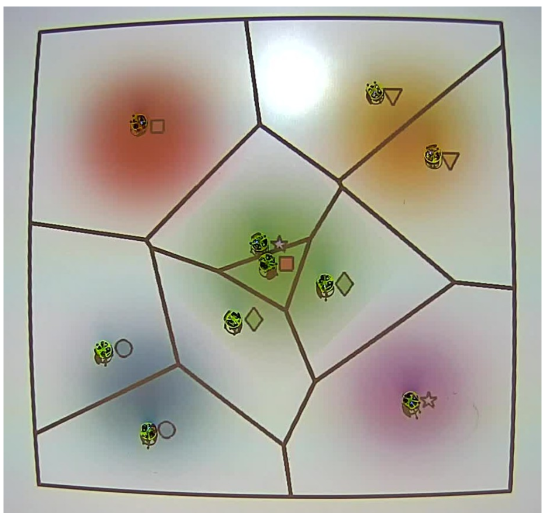
[18] D. Pickem, P. Glotfelter, L. Wang, M. Mote, A. Ames, E. Feron, and M. Egerstedt, "The Robotarium: A remotely accessible swarm robotics research testbed," in *2017 IEEE International Conference on Robotics and Automation (ICRA)*, Singapore, May 2017, pp. 1699–1706.



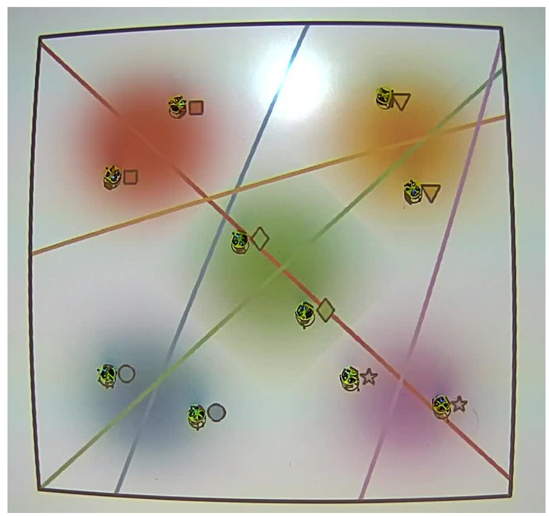
(a) Without communications, Exp. 2.



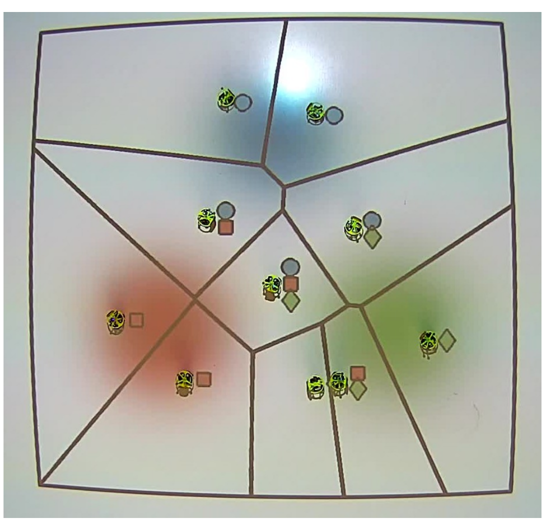
(b) Proposed algorithm, with communications, Exp. 2.



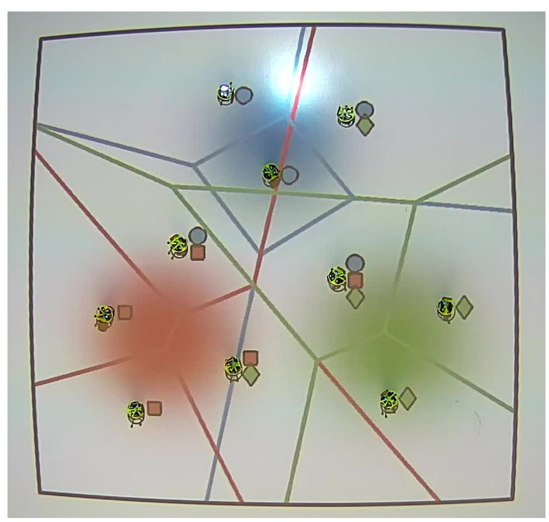
(c) Without communications, Exp. 4.



(d) Proposed algorithm, with communications, Exp. 4.



(e) Without communications, Exp. 6.



(f) Proposed algorithm, with communications, Exp. 6.

Fig. 3. Final configurations of the multi-agent team for Experiments 2, 4 and 6. Figures (a), (c) and (e) correspond to the coverage control algorithm without communications in [13], while Figs. (b), (d) and (e) depict the final spatial allocation of the team when running the proposed algorithm, which includes communications, for the same sensor configurations and initial conditions. The parts of the domain shaded with the different colors represent the areas of highest density, with each color identifying a different sensing modality. Each robot has a collection of symbols located to its right, which represents the its sensor equipment and its color coded according to the corresponding density functions.With respect to the specifications in Table I, the colors –blue, red, green, orange, and purple – correspond to the numerical identifiers 1 to 5 in the sensor equipments.